Universal Transmission and Combining for Ultra-Reliable MIMO Relaying

Barak Avraham, Elad Domanovitz, Uri Erez
School of Electrical Engineering
Tel Aviv University, Tel Aviv, Israel
Email: {domanovi, uri}@eng.tau.ac.il

2024 IEEE International Symposium on Information Theory (ISIT) | 979-8-3503-8284-6/24/$31.00 ©2024 IEEE | DOI: 10.1109/ISIT57864.2024.10619281

***Abstract*—We propose a novel transmission scheme for ultra-reliable multi-hop multiple-antenna communication where relays perform only universal linear operations on the received signals. In particular, the operations are channel-oblivious, and no detection takes place in intermediate relaying nodes. The processing at each relay may be viewed as a concatenation of a dimension-reduction operation, i.e., a universal combining, and orthogonal space-time block coding, i.e., a universal transmission operation. It is demonstrated that the developed transmission-combining relaying technique guarantees reliable communication in a very strong sense: so long as all relay-to-relay links have a non-vanishing capacity, reliable communication is possible. The proposed schemes are derived by establishing a certain operational equivalence relationship between the true channel and an associated multiple-input single-output channel.**

I. Introduction and Background

Ultra-reliable communication over multi-hop networks has become an essential component for many applications. Among them, we can note reliable cloud (edge) connectivity and offloading [1], vehicle-to-vehicle (V2V) wireless coordination [2], mission-critical machine-type communications [3] and many others (see, e.g., [4]).

A key challenge for achieving the goal of ultra-reliable communication over wireless links is overcoming channel uncertainty, especially so when small payloads are to be transmitted. One of the fundamental approaches to enhancing robustness to channel variability is the use of multiple transmit and/or receive antennas, thus achieving diversity.

The difficulty of achieving full diversity greatly depends on two factors: the availability of channel state information (CSI) and the MIMO configuration. Clearly, the task is simple if CSI is available at all nodes, allowing one to adapt the transmission rate accordingly so that an outage does not arise. Moreover, by employing the singular value decomposition, simple coding and detection can be used.

Achieving full diversity for a single hop transmission when CSI is available only at the receiver on a MIMO channel is also a well-studied problem. It can be accomplished via space-time coding, and in particular, via orthogonal space-time coding [5]. The latter method will play a central role in the sequel.

In this work, we consider ultra-reliable transmission over a MIMO *multi-hop* scenario (Figure 1) where we wish to guarantee robust transmission over the end-to-end link, between a source node and one of several possible destination nodes (where the specific destination is not known in advance). Hence, in such a setting, selection methods (see, e.g. [2], [6], [7]) are inapplicable.

A straightforward approach for channel-oblivious multi-hop relaying (i.e., with no CSI at the relays) is that of amplify-and-forward (AF), i.e., each relay retransmits the received signal (plus noise) to the next relay node, amplified to meet some signal power constraint. However, as we demonstrate in the sequel, two "ill-matched" MIMO channels (3), each having adequate capacity, may nonetheless result in losing the signal entirely at the destination (zero capacity). Hence, AF is ill-suited for ultra-reliable multi-hop transmission.

Furthermore, straightforward extensions which are DMT optimal (assuming Rayleigh fading) for both single and multi-hop, see e.g. [8], fail on the same example (two ill-matched MIMO channels) as AF above. See Remark 2.

Despite the fact that the above example may be viewed as a rare event, when considering ultra-reliable applications it is evident that transmission schemes that are robust to any combination of MIMO channels are desirable. That is, we seek schemes that guarantee successful transmission, so long as all individual links are of "adequate" quality.

Schemes meeting such stringent requirements, formally defined in the sequel, will be called universal, ultra-reliable transmission schemes. Designing such schemes while simultaneously retaining a high symbol rate (i.e., bandwidth (BW) efficiency) is a non-trivial task and is the goal of this work.[1]

The simplest AF method that achieves ultra-reliability is that of repetition. Assuming a cascade of nodes, each equipped with N_t transmit, and N_r receive antennas, we show in the sequel that the BW efficiency over the ℓ-th hop is $\frac{1}{N_t} \cdot \frac{1}{(N_t \cdot N_r)^{\ell-1}}$ which decreases exponentially with factor $N_t \cdot N_r$ in ℓ.

Building upon orthogonal transmission-combining schemes presented in [9] and [10], we propose novel transmit and combining methods that achieve high BW efficiency over the multiple links and scale much better in the number of hops ℓ.

II. System Model

We analyze an ad-hoc communication scenario where a single source wishes to transmit a message to a group of

This work was supported by the ISRAEL SCIENCE FOUNDATION (ISF), grant No. 588/19 and 736/23.

[1]We use the terms symbol rate and BW efficiency interchangeably.

possible destinations via relays. For simplicity, we may assume each group of relays is located within the same distance from the transmitter. We denote the source node by $D^{(1)}$, the relays at group ℓ by $D_{i_\ell}^{(\ell)}$ and the destination nodes by $D_{i_{L+1}}^{(L+1)}$. The number of relay nodes can be arbitrary, and thus, the range of the index i_ℓ can also be arbitrarily large. We assume full synchronization in the network and ignore propagation delay. We further assume node $D_{i_\ell}^{(\ell)}$ can only transmit information to nodes $\{D_{i_{\ell+1}}^{(\ell+1)}\}$ and receive information from nodes $\{D_{i_{\ell-1}}^{(\ell-1)}\}$. We further assume each node $D_{i_\ell}^{(\ell)}$ receives information only from a single node in $\{D_{i_\ell-1}^{(\ell-1)}\}$, i.e., there is no interference from adjacent transmitting nodes (and in fact, we define a tree topology). Transmission from the source to a single destination can be viewed as a line network. We note that the index i_ℓ, identifying a specific node within group ℓ plays no role in the analysis.

While in general, we may assume a different number of transmit/receive antennas for each node, in this work, we assume that the source has N_t antennas, the destination has N_r antennas, and each relay has N_t transmit and N_r receive antennas. We denote the channel between adjacent nodes in layer ℓ and $\ell+1$ by $\mathbf{H}^{(\ell,i_\ell)} \in \mathbb{C}^{N_t \times N_r}$, and the channel coefficient from transmit antenna m in node ℓ to receive antenna n in node $\ell+1$ by $h_{mn}^{(\ell,i_\ell)}$.

In this work, we assume, for simplicity, that all channels in group ℓ have the same Frobenius norm, which is strictly positive, i.e., $\|\mathbf{H}^{(\ell,i_\ell)}\|_F^2 = \|\mathbf{H}^{(\ell)}\|_F^2 > 0$ for all i_ℓ. As all channels have the same Frobenius norm, the main challenge we address is the possibility that the channel matrices are *ill-matched*, a notion described below. Our goal is to guarantee successful transmission under the worst-case conditions of an end-to-end line-network, from source to some destination, as depicted by the highlighted blocks in Figure 1.

We describe each relay node as composed of two functional blocks: a combining block and a transmission block which is depicted in Figure 2. We assume that each transmitter block uses a space-time block code (STBC), that is, the transmitter in node ℓ wishes to transmit $k^{(\ell)}$ information symbols $\mathbf{s}^{(\ell)} = \left[s_1^{(\ell)}, \dots s_{k^{(\ell)}}^{(\ell)}\right]^T$ where $s_w^{(\ell)} \in \mathbb{C}$, over $T^{(\ell)}$ channel uses.[2] We describe the transmitted symbols from node ℓ, $\ell \in \{1, \dots, L\}$ by a STBC, denoted by $\mathbf{C}^{(\ell)}(\mathbf{s}^{(\ell)}) \in \mathbb{C}^{T^{(\ell)} \times N_t}$, where the entry $x_{t,m}^{(\ell)}$ is the transmitted symbol at time t from antenna m, $t \in [1, \dots, T^{(\ell)}]$ and $m \in [1, \dots, N_t]$.

We assume a sum power constraint, i.e., $\frac{1}{N_t \cdot T^{(\ell)}} \sum_{t=1}^{T^{(\ell)}} \mathbb{E}\left[\|x_{t,m}^{(\ell)}\|^2\right] = P$.

It is assumed that the channel remains fixed during the transmission duration. We denote the received signal at node $\ell+1$ as

$$\mathbf{Y}^{(\ell+1)} = \mathbf{C}^{(\ell)}(\mathbf{s}^{(\ell)})\mathbf{H}^{(\ell)} + \mathbf{Z}^{(\ell+1)}. \tag{1}$$

where $\mathbf{Z}^{\ell+1} \in \mathbb{C}^{T^{(\ell)} \times N_r}$ is the space-time noise matrix where $z_{t,n}^{(\ell+1)}$ is the noise at each received antenna n at node $\ell+1$ at time t which we assume is i.i.d., circularly-symmetric complex Gaussian noise with power $1/N_r$.

[2]In this paper, the vertical “direction” corresponds to the time axis.

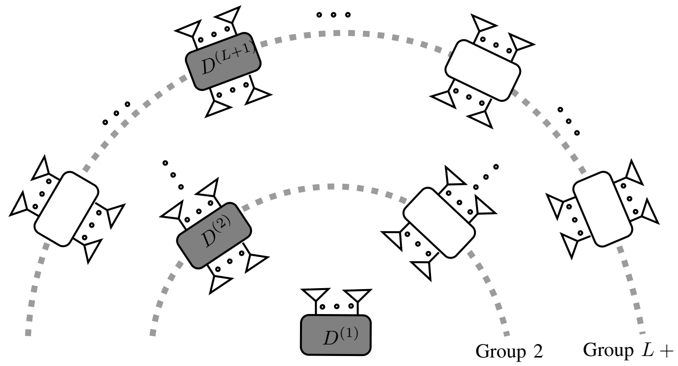

Figure 1. Multi-hop MIMO relaying.

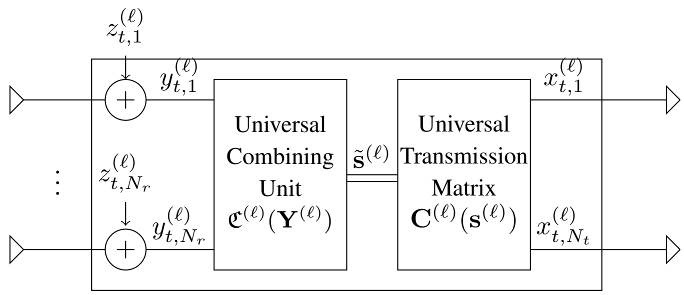

Figure 2. Zoom into node ℓ.

The induced transmission symbol rate (which is the transmission BW efficiency for transmission of $k^{(\ell)}$ information symbols) when using the OSTBC $C^{(\ell)}(\mathbf{s}^{(\ell)})$ is $R_s^{(\ell)} \triangleq \frac{k^{(\ell)}}{T^{(\ell)}}$.

Remark 1. *While in the present context, $T^{(\ell)}$ refers to the number of time slots used, it may equally refer to frequency slots or, more generally, degrees of freedom.*

The combining block takes the matrix $\mathbf{Y}^{(\ell)}$ as its input and returns as output a column vector $\tilde{\mathbf{s}}^{(\ell)} \in \mathbb{C}^{k^{(\ell)}}$, representing a stream of symbols that could be used by the destination to recover the original information symbols, or alternatively, relayed efficiently over another MIMO hop.[3] The essence of the combining unit is that it performs dimension reduction. Thus, it may improve the overall bandwidth efficiency.

We limit our interest to combining blocks that perform linear or widely-linear operations, denoted by $\mathfrak{C}^{(\ell)}(\cdot)$ over $\mathbf{Y}^{(\ell)}$, resulting in the vector $\tilde{\mathbf{s}}^{(\ell)} \in \mathbb{C}^{k^{(\ell)}}$. [4]

The reciprocal of the expansion rate of information symbols in node ℓ (relative to the number of information symbols sent from node $\ell-1$) induced by the combining unit is denoted by $\gamma^{(\ell)} \triangleq \frac{k^{(\ell-1)}}{k^{(\ell)}}$.

Therefore, the overall BW efficiency of a scheme at node ℓ

[3]$\tilde{\mathbf{s}}^{(\ell)}$ is used instead of $\mathbf{s}^{(\ell)}$ to highlight that it is not necessarily equal to the original information symbols sent from the source.

[4]Widely linear operations are linear operations when viewed over the reals.

is denoted as

$$\eta^{(\ell)} = \frac{k^{(1)}}{T^{(\ell)}} = \left(\prod_{i=2}^{\ell} \frac{k^{(i-1)}}{k^{(i)}}\right) \cdot \frac{k^{(\ell)}}{T^{(\ell)}} = R_s^{(\ell)} \cdot \prod_{i=2}^{\ell} \gamma^{(i)}. \quad (2)$$

Finally, we assume that the destination node is the only one that has access to CSI, i.e., performs channel estimation. Further, it can be shown that the destination node need only to estimate products of channel coefficients that form a source-to-destination path.

The transmission-combining schemes we propose all result in an effective end-to-end unitary MIMO channel. The destination can recover the data by inverting the channel with no noise amplification and then applying a scalar AWGN decoder.

A. Notations and basic properties

We use bold capital letters to denote matrices and bold small letters to denote vectors. Assuming matrix $\mathbf{X}$ is composed of N column vectors $\mathbf{X} = \begin{bmatrix}\mathbf{x}_1 & \cdots & \mathbf{x}_N\end{bmatrix}$ we denote $\mathbf{X}_{\text{vect}} \triangleq \begin{bmatrix}\mathbf{x}_1^T & \cdots & \mathbf{x}_N^T\end{bmatrix}^T$. For a complex vector $\mathbf{x}$ we define its real representaiton as $\mathbf{x}_r \triangleq \begin{bmatrix}\Re[\mathbf{x}]^{\mathbf{T}} & \Im[\mathbf{x}]^{\mathbf{T}}\end{bmatrix}^T$.

Let $\mathcal{H}$ be an $m \times n$ semi-unitary matrix (i.e., $\mathcal{H}^H\mathcal{H} = \mathbf{I}_{n\times n}$). Clearly, the following holds.

Property 1. *The product of semi-unitary matrices is also a semi-unitary matrix, i.e. if $\mathcal{H}_1$ is an $m_1 \times n_1$ semi-unitary matrix and $\mathcal{H}_2$ is an $n_1 \times n_2$ semi-unitary matrix, then $\mathcal{H}_1\mathcal{H}_2$ is an $m_1 \times n_2$ semi-unitary matrix.*

Property 2. *Let $\mathbf{z}$ be an i.i.d., circularly-symmetric complex Gaussian vector with entries of power P. Let $\tilde{\mathbf{z}} = \mathcal{H}\mathbf{z}$ Then $\tilde{\mathbf{z}}$ is also an i.i.d., circularly-symmetric complex Gaussian vector (of smaller dimension) with entries of power P.*

III. Schemes for 2×2 MIMO Channels

We begin by describing a few AF schemes for a line network with $N_r = N_t = 2$ in all the nodes.

A. Space-Only Amplify-and-Forward

To simplify the derivations in this case, we describe the scheme in the absence of noise.[5] Each relay retransmits the symbols received from the previous node, i.e., $\mathbf{C}^{(\ell)}(\tilde{\mathbf{s}}^{(\ell)}) = \tilde{\mathbf{s}}^{(\ell)}$. Next, we explicitly describe the case of $L = 2$.

- Transmission from the source: $\mathbf{C}^{(1)}(\mathbf{s}^{(1)}) = \begin{bmatrix}s_1 & s_2\end{bmatrix}$
- Node 2:
  - Input: $\mathbf{y}^{(2)} = \mathbf{C}^{(1)}(\mathbf{s}^{(1)})\mathbf{H}^{(1)}$
  - Combining: $\tilde{\mathbf{s}}^{(2)} = \mathfrak{C}^{(2)}(\mathbf{Y}^{(2)}) = \mathbf{Y}^{(2)}$
  - Transmission: $\mathbf{C}^{(2)}(\tilde{\mathbf{s}}^{(2)}) = \tilde{\mathbf{s}}^{(2)} = \mathbf{Y}^{(2)}$
- Node 3: $\mathbf{y}^{(3)} = \mathbf{C}^{(2)}(\tilde{\mathbf{s}}^{(2)})\mathbf{H}^{(2)} = \mathbf{C}^{(1)}(\mathbf{s}^{(1)})\mathbf{H}^{(1)}\mathbf{H}^{(2)}$.

Since we have $\mathbf{C}^{(\ell)}(\mathbf{s}^{(\ell)}) = \mathbf{s}^{(\ell)}$ it means that $T^{(\ell)} = 1$ and $k^{(\ell)} = 2$ and therefore $R_s^{(\ell)} = 2$. Further, since $\mathfrak{C}^{(\ell)}(\mathbf{Y}^{(\ell)}) = \mathbf{Y}^{(\ell)}$, we have also $\gamma^{(\ell)} = 1$. Hence, the overall efficiency at each node ℓ, $\ell \in \{2, \ldots, L+1\}$ is $\eta^{(\ell)} = 2$ (i.e. no BW loss).

However, consider the following two channel matrices

$$\mathbf{H}^{(1)} = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\ \mathbf{H}^{(2)} = \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix},\ \mathbf{H}^{(1)}\mathbf{H}^{(2)} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}. \quad (3)$$

[5]We may also disregard the power constraint.

Clearly, we cannot recover any of the transmitted symbols at destination D_3. Hence, the scheme does not achieve the goal of ultra-reliability. We refer to such channel matrices as “ill-matched".

Remark 2. *We note that DMT-optimal relaying schemes also do not gurantee successful transmission over such channels. The reason is that in order for a scheme to be diversity-optimal for a MIMO multihop line network, under the assumption of i.i.d. Rayleigh fading, the scheme need only guarantee successful transmission so long as $N_t \times N_r$ (specific, chosen a priori) disjoint paths in the edge graph between the source and destination remain intact [8]. In contrast, the notion of ulra-reliability we advocate requires robustness to any such failure. That is, successful transmission should be guaranteed so long as there remains a single end-to-end path intact.*

B. Repetition

When using repetition diversity, the transmission matrix from the source is given by $\mathbf{C}^{(1)}(s_1) = s_1 \otimes \mathbf{I}_{2\times 2}$, and from node $\ell \geq 2$ as

$$\mathbf{C}^{(\ell)}(\tilde{\mathbf{s}}^{(\ell)}) = \alpha\tilde{\mathbf{s}}^{(\ell)} \otimes \mathbf{I}_{2\times 2}, \quad (4)$$

where $\alpha = \sqrt{\frac{P}{1+P}}$ is a power normalization factor. We note that, in general, transmitting $\mathbf{C}^{(\ell-1)}(\tilde{\mathbf{s}}^{(\ell-1)})$ (which is composed of $k^{(\ell-1)}$ information symbols) from node $\ell - 1$, results in

$$\mathbf{Y}^{(\ell)} = \alpha\left(\tilde{\mathbf{s}}^{(\ell-1)} \otimes \mathbf{I}_{2\times 2}\right)\mathbf{H}^{(\ell-1)} + \mathbf{Z}^{(\ell+1)},$$

which can be expressed using an equivalent virtual channel matrix (EVCM) as

$$\check{\mathbf{y}}^{(\ell)} = \alpha \underbrace{\mathbf{H}_{\text{vect}}^{(\ell-1)} \otimes \mathbf{I}_{k^{(\ell-1)}\times k^{(\ell-1)}}}_{\mathcal{H}(\mathbf{H}^{(\ell-1)})} \tilde{\mathbf{s}}^{(\ell-1)} + \check{\mathbf{z}}^{(\ell)}, \quad (5)$$

where $\mathcal{H}(\mathbf{H}^{(\ell-1)})$ is a scaled semi-unitary matrix. We note that since the entries of the vector $\check{\mathbf{z}}^{(\ell)}$ are identical to those of the matrix $\mathbf{Z}_{\text{vect}}^{(\ell)}$, it is a complex Gaussian vector with i.i.d., circularly-symmetric with entries of power $1/N_r = 1/2$.

We assume that the combining operation results in $\tilde{\mathbf{s}}^{(\ell)} = \check{\mathbf{y}}^{(\ell)}$ (which is subsequently fed to the transmission block (4)) which amounts to some permutation of $\mathbf{Y}_{\text{vect}}^{(\ell)}$.

An explicit description of the transmission and combining of a line network with three nodes appears in Appendix A. The following Lemma (whose proof is in Appendix A) shows that ultra-reliability over 2×2 multi-hop channels is achieved when using transmission (4) and combining (5) in all nodes.

Lemma 1. *Assuming $\|\mathbf{H}^{(\ell)}\|_F^2 > 0$, $\forall \ell \in \{1, \ldots, L\}$, and using transmission matrix* (4) *and combining operation* (5) *in all nodes, then node $L+1$, using CSI, can convert the end-to-end MIMO channel to the following SISO channel,*

$$\hat{s}_1^{L+1} = \alpha^{L-1}\prod_{\ell=1}^{L}\|\mathbf{H}^{(\ell)}\|_F \cdot s_1 + \alpha^{L-1}\prod_{\ell=2}^{L}\|\mathbf{H}^{(\ell)}\|_F\hat{z}^{(3)} + \alpha^{L-2}\prod_{\ell=3}^{L}\|\mathbf{H}^{(\ell)}\|_F\hat{z}^{(2)} + \cdots + \hat{z}^{(L+1)} \quad (6)$$

where $\alpha = \sqrt{\frac{P}{1+P}}$ and $\hat{z}^{(\ell)}$ are independent, circularly-symmetric complex Gaussian noise with power $1/N_r = 1/2$.

Unlike the case of space-only AF, it is clear from (6) that repetition transmission *achieves the goal of ultra-reliable communication* since the effective channel gain, being the product of the individual Frobenius norms of the channels, is strictly positive.

The downside is that very poor BW efficiency is achieved. Specifically, by (2), we have $\eta^{(\ell)}_{\text{Rep+Rep}} = \left(\frac{1}{N_r N_t}\right)^{\ell-1}\left(\frac{1}{N_t}\right)$.

C. Alamouti + repetition

The Alamouti transmission block for two information symbols is defined as

$$C_{\text{Ala}}\left(\begin{bmatrix} s_1 & s_2\end{bmatrix}^T\right) \triangleq \frac{1}{\sqrt{2}}\begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^*\end{bmatrix}. \tag{7}$$

The source node D_1 uses (7) for transmission. Assuming an even number of information symbols to be sent, the transmission matrix for $\ell \geq 2$ is defined as

$$\mathbf{C}^{(\ell)}(\tilde{\mathbf{s}}^{(\ell)}) = \sqrt{\frac{P}{1+P}}\begin{bmatrix} C_{\text{Ala}}\left(\begin{bmatrix}\tilde{s}_1^{(\ell)} & \left(\tilde{s}_2^{(\ell)}\right)^*\end{bmatrix}^T\right) \\ \vdots \\ C_{\text{Ala}}\left(\begin{bmatrix}\tilde{s}_{k^{(\ell)}-1}^{(\ell)} & \left(\tilde{s}_{k^{(\ell)}}^{(\ell)}\right)^*\end{bmatrix}^T\right)\end{bmatrix}. \tag{8}$$

We note that when transmitting (8) , $\mathbf{Y}^{(\ell)}$ can be expressed using the EVCM as

$$\check{\mathbf{y}}^{(\ell)} = \sqrt{\frac{P}{2(1+P)}}\left[\mathbf{I}_{2^{\ell-1}\times 2^{\ell-1}} \otimes \mathcal{H}_{\text{Ala}}(\mathbf{H}^{(\ell-1)})\right]\tilde{\mathbf{s}}^{(\ell-1)} + \check{\mathbf{z}}^{(\ell)} \tag{9}$$

where $\left[\mathbf{I}_{2^{\ell-1}\times 2^{\ell-1}} \otimes \mathcal{H}_{\text{Ala}}(\mathbf{H}^{(\ell-1)})\right]$ is a scaled semi-unitary matrix and $\check{\mathbf{z}}^{(\ell)}$ is a complex Gaussian vector with i.i.d., circularly-symmetric entries of power $1/N_r = 1/2$. The combining operation amounts to obtaining (9), i.e. $\tilde{\mathbf{s}}^{(\ell)} = \check{\mathbf{y}}^{(\ell)}$.

An explicit example of a line network with three nodes is detailed in Appendix B. The following Lemma (whose proof is in Appendix B) shows that ultra-reliability over 2×2 multi-hop channels is achieved when using transmission (8) and combining (9) in all nodes.

Lemma 2. *Assuming $\|\mathbf{H}^{(\ell)}\|_F^2 > 0$, $\forall \ell\{1,\ldots,L\}$, using transmission matrix (8) and combining operation (9) in all nodes, then node $L+1$, using CSI, can convert the end-to-end MIMO channel to the following i.i.d. SISO channels,*

$$\begin{bmatrix}\hat{s}_1^{(L+1)} \\ \hat{s}_2^{(L+1)}\end{bmatrix} = \frac{\alpha^{L-1}}{\sqrt{2}}\prod_{\ell=1}^{L}\|\mathbf{H}^{(\ell)}\|_F\begin{bmatrix}s_1 \\ s_2\end{bmatrix} + \alpha^{L-1}\prod_{\ell=2}^{L}\|\mathbf{H}^{(\ell)}\|_F\hat{\mathbf{z}}^{(2)}$$
$$+\,\alpha^{L-2}\prod_{\ell=3}^{L}\|\mathbf{H}^{(\ell)}\|_F\hat{\mathbf{z}}^{(3)} + \ldots + \hat{\mathbf{z}}^{(L+1)}, \tag{10}$$

where $\alpha = \sqrt{\frac{P}{2(1+P)}}$ and where $\hat{\mathbf{z}}^{(\ell)}$ are vectors of i.i.d., circularly-symmetric complex Gaussian noise where the power of $\hat{\mathbf{z}}_i^{(\ell)}$ is $1/N_r = 1/2$.

Transmission Alamouti (8) and generating the EVCM to be transmitted to the next node (9) results in $\eta^{(\ell)}_{\text{Ala+Rep}} = \frac{1}{2}^{(\ell-1)}$.

D. Universal Transmission-Combining

The concept of universal combining was presented in [9]. We first briefly describe how to convert the 2×2 MIMO channel to the equivalent of transmission of a rate $3/4$ OSTBC over a 4×1 MISO channel channel. Let the transmission matrix be

$$\mathbf{C}_{3/4}\left(\begin{bmatrix}s_1 & s_2 & s_3\end{bmatrix}^T\right) = \sqrt{\frac{6}{5}}\begin{bmatrix} s_1^* & s_2^* \\ -s_2 & s_1 \\ s_3 & 0 \\ 0 & s_3 \\ s_1^* & -s_2 \\ s_2^* & s_1\end{bmatrix}. \tag{11}$$

For a single channel ($L = 2$), the received matrix $\mathbf{Y}^{(2)}$ is $\mathbf{Y}^{(2)} = \mathbf{C}_{3/4}\left(\begin{bmatrix}s_1 & s_2 & s_3\end{bmatrix}^T\right)\mathbf{H}^{(1)} + \mathbf{Z}^{(2)}$.

Let the combining matrix be

$$\mathfrak{C}_{3/4}\left(\mathbf{Y}^{(2)}\right) = \frac{1}{\sqrt{2}}\begin{bmatrix}\left(Y_{1,1}^{(2)}\right)^* + Y_{3,2}^{(2)} \\ \left(Y_{2,1}^{(2)}\right)^* + Y_{4,2}^{(2)} \\ \left(-Y_{3,1}^{(2)}\right)^* + Y_{5,2}^{(2)} \\ \left(-Y_{4,1}^{(2)}\right)^* + Y_{6,2}^{(2)}\end{bmatrix}. \tag{12}$$

This results in

$$\mathfrak{C}(\mathbf{Y}^{(2)}) = \sqrt{\frac{3}{5}}\underbrace{\begin{bmatrix} s_1 & s_2 & s_3 & 0 \\ -s_2^* & s_1^* & 0 & s_3 \\ -s_3^* & 0 & s_1^* & -s_2 \\ 0 & -s_3^* & s_2^* & s_1\end{bmatrix}}_{\mathbf{C}_{\text{MISO},3/4}\left(\begin{bmatrix}s_1 & s_2 & s_3\end{bmatrix}^T\right)}\begin{bmatrix}\left(h_{1,1}^{(1)}\right)^* \\ \left(h_{2,1}^{(1)}\right)^* \\ h_{1,2}^{(1)} \\ h_{2,2}^{(1)}\end{bmatrix} + \tilde{\mathbf{z}}^{(2)}.$$

This relation, obtained by the transmission-combining scheme, is precisely the input and output relation when transmitting the rate $3/4$ OSTBC over a 4×1 MISO channel. Although this OSTBC does not have an EVCM over the complex field, it does have an EVCM over the reals (see Section 1.2.2 in [11]), leading to the EVCM relation $\check{\mathbf{y}}_r^{(2)} = \check{\mathbf{W}}^{(1)}\begin{bmatrix}s_1 & s_2 & s_3\end{bmatrix}_r^T + \check{\mathbf{z}}_r^{(2)}$, where $\check{\mathbf{W}}^{(1)}$ is a semi-orthogonal matrix. Assuming node D_2 has access to CSI it can estimate

$$\hat{\mathbf{s}}_r^{(2)} = \frac{\left(\check{\mathbf{W}}^{(1)}\right)^T}{\|\mathbf{H}^{(1)}\|_F}\check{\mathbf{y}}_r^{(2)} = \sqrt{\frac{3}{5}}\|\mathbf{H}^{(1)}\|_F\begin{bmatrix}s_1 & s_2 & s_3\end{bmatrix}_r^T + \hat{\mathbf{z}}_r^{(2)},$$

where following Property 2, $\hat{\mathbf{z}}_r^{(2)}$ is real i.i.d. Gaussian noise where each entry has power of $1/4$.

In general, we assume $\tilde{\mathbf{s}}^{(\ell)}$ contains $k^{(\ell)}$ information symbols such that $\text{mod}(k^{(\ell)}, 3) = 0$.[6] The transmission matrix is defined as

$$\mathbf{C}^{(\ell)}\left(\tilde{\mathbf{s}}^{(\ell)}\right) = \gamma(\ell)\begin{bmatrix}\mathbf{C}_{3/4}\left(\begin{bmatrix}\tilde{s}_1^{(\ell)} & \tilde{s}_2^{(\ell)} & \tilde{s}_3^{(\ell)}\end{bmatrix}^T\right) \\ \vdots \\ \mathbf{C}_{3/4}\left(\begin{bmatrix}\tilde{s}_{k^{(\ell)}-2}^{(\ell)} & \tilde{s}_{k^{(\ell)}-1}^{(\ell)} & \tilde{s}_{k^{(\ell)}}^{(\ell)}\end{bmatrix}^T\right)\end{bmatrix}, \tag{13}$$

[6]Transmitting $k^{(1)} = 3^{L-1}$ information symbols from the transmitter ensures that in all nodes $\text{mod}(k^{(\ell)}, 3) = 0$.

where $\gamma(\ell) = \begin{cases} \sqrt{\frac{P}{1+P}} & \ell > 1 \\ 1 & \ell = 1 \end{cases}$. In general, node ℓ receives

$$\mathbf{Y}^{(\ell)} = \mathbf{C}^{(\ell-1)}\left(\tilde{\mathbf{s}}^{(\ell-1)}\right)\mathbf{H}^{(\ell-1)} + \mathbf{Z}^{(\ell)} \triangleq \begin{bmatrix} \mathbf{Y}_1^{(\ell)} \\ \vdots \\ \mathbf{Y}_{k^{(\ell-1)}/3}^{(\ell)} \end{bmatrix}$$

where $\mathbf{Y}_{j^{(\ell)}}^{(\ell)}$, $j^{(\ell)} \in \{1,\ldots,k^{(\ell-1)}/3\}$ is a 6×2 matrix which is the outcome of transmitting $\mathbf{C}_{3/4}\left(\begin{bmatrix}\tilde{s}_{3j^{(\ell)}-2}^{(\ell-1)} & \tilde{s}_{3j^{(\ell)}-1}^{(\ell-1)} & \tilde{s}_{3j^{(\ell)}}^{(\ell-1)}\end{bmatrix}^T\right)$. We define the general transformation that converts the MIMO channel to an equivalent MISO channel as

$$\mathfrak{C}^{(\ell)}\left(\mathbf{Y}^{(\ell)}\right) = \left[\mathfrak{C}_{3/4}\left(\mathbf{Y}_1^{(\ell)}\right) \quad \cdots \quad \mathfrak{C}_{3/4}\left(\mathbf{Y}_{k^{(\ell-1)}/3}^{(\ell)}\right)\right]^T. \tag{14}$$

which converts $\mathbf{Y}^{(\ell)}$ to

$$\begin{bmatrix} \mathbf{C}_{\text{MISO},3/4}\left(\begin{bmatrix}\tilde{s}_1^{\ell-1} & \tilde{s}_2^{\ell-1} & \tilde{s}_3^{\ell-1}\end{bmatrix}^T\right) \\ \vdots \\ \mathbf{C}_{\text{MISO},3/4}\left(\begin{bmatrix}\tilde{s}_{k^\ell-2}^{\ell-1} & \tilde{s}_{k^\ell-1}^{\ell-1} & \tilde{s}_{k^\ell}^{\ell-1}\end{bmatrix}^T\right) \end{bmatrix} \begin{bmatrix} \left(h_{1,1}^{(\ell-1)}\right)^* \\ \left(h_{2,1}^{(\ell-1)}\right)^* \\ h_{1,2}^{(\ell-1)} \\ h_{2,2}^{(\ell-1)} \end{bmatrix} + \mathfrak{C}^{(\ell)}(\mathbf{Z}^{(\ell)}).$$

We note that since each entry in $\mathfrak{C}^{(\ell)}(\mathbf{Z}^{(\ell)})$ is composed of a (normalized) summation of distinct entries from $\mathbf{Z}^{(\ell)}$, it follows that the entries are AWGN with power $1/2N_r = 1/4$. This can be expressed over the reals through rearranging the real representation of $\mathfrak{C}(\mathbf{Y}^{(\ell)})$ as

$$\breve{\mathbf{y}}_r^{(\ell)} = \breve{\mathbf{W}}_r^{(\ell)}\breve{\mathbf{s}}_r^{(\ell-1)} + \breve{\mathbf{z}}_r^{(\ell)}. \tag{15}$$

where $\breve{\mathbf{W}}$ is a semi-orthogonal matrix. We assume that the combining block in node ℓ transfers $\tilde{\mathbf{s}}^\ell = \breve{\mathbf{y}}^{(\ell)}$ to the transmission block.

We next establish the following lemma (whose proof is in Appendix C).

Lemma 3. *Assuming* $\|\mathbf{H}^{(\ell)}\|_F^2 > 0$, $\forall \ell\{1,\ldots,L\}$, *then using the transmission matrix* (13) *and combining operation that results in* (15). *Further, at node* $L+1$, *the end-to-end MIMO channel can be converted to the following i.i.d. SISO channels,*

$$\hat{s}_i^{(L+1)} = \alpha^{L-1}\left(\sqrt{\frac{3}{5}}\right)\prod_{\ell=1}^{L}\|\mathbf{H}^{(\ell)}\|_F \cdot s_i + \alpha^{L-1}\prod_{\ell=2}^{L}\|\mathbf{H}^{(\ell)}\|_F\hat{z}_i^{(2)}$$
$$+ \alpha^{L-2}\prod_{\ell=3}^{L}\|\mathbf{H}^{(\ell)}\|_F\hat{z}_i^{(3)} + \cdots + \hat{z}_i^{(L+1)}, \tag{16}$$

where $i \in \{1,\ldots,k^{(1)}\}$, $\alpha = \sqrt{\frac{3P}{5(1+P)}}$ *and where* $\hat{\mathbf{z}}^{(\ell)}$ *are vectors of i.i.d., circularly-symmetric complex Gaussian noise where the power of* $\hat{\mathbf{z}}_i^{(\ell)}$ *is* $1/N_r = 1/2$.

Using the transmission operation (13) and the combining opertation (14), results in BW efficiency of $\eta_{3/4}^{(\ell)} = \frac{1}{2}\left(\frac{3}{4}\right)^{\ell-1}$.

E. Comparison of different schemes for 2×2 channels

The upper sub-figure of Figure 3 depicts the efficiency of the different schemes, all of which guarantee ultra-efficient multi-hop MIMO 2×2 relaying. It can be seen that the rate-$3/4$ OSTBC based transmission combining scheme has the best efficiency for networks with three or more hops.

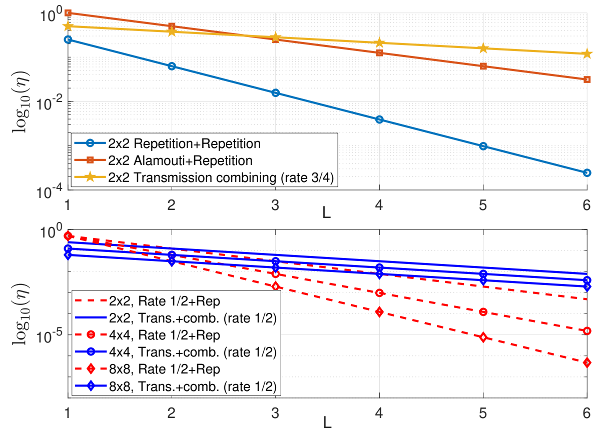

Figure 3. Comparing the BW efficiency of different schemes.

IV. Schemes for General $N \times N$ MIMO Channels

Unfortunately, there is no rate 1 OSTBC for more than two transmit antennas. However, there exists a rate of $1/2$ for any number of antennas [5]. Hence, we consider these as a benchmark (as it clearly outperforms repetition).

In general, the number of information symbols sent by a single real OSTBC of rate 1 is given by the Hurwitz-Radon number [5], which we denote as $\rho(N)$. Transmitting $\rho(N)$ complex information symbols from N transmit antennas at rate $1/2$ is achieved with $T = 2\rho(N)$.

Hence, transmission at node ℓ has $R_s^{(\ell)} = 1/2$. The combining block transmits each received symbol from all transmit antennas. We therfore have $\gamma^{(\ell)} = \frac{\rho(N)}{2\cdot\rho(N)\cdot N}$. It follows that $\eta_{Rate_{1/2}+Rep.} = \frac{1}{2}\left(\frac{1}{2N}\right)^{\ell-1}$.

For an $N \times N$ channel, we apply transmission combining that converts the channel to resemble a MISO channel with N^2 transmit antennas. One option for a transmission scheme first transmits each information symbol from each antenna (while other antennas are silent) and then its conjugate (while other antennas are silant). Since the OSTBC for a MISO channel is composed only of single information symbols and their conjugates, it immediately follows that a combining matrix exists that generates an equivalent channel. An example for 2×2 channels appears in Appendix D.

We therefore have $R_s^{(\ell)} = \frac{\rho(N^2)}{\rho(N^2)\cdot N\cdot 2}$ It can be shown that we have $\gamma^{(\ell)} = \frac{1}{2}$. This means that $\eta_{Trans.+Comb.} = \frac{1}{2N}\left(\frac{1}{2}\right)^{\ell-1}$.

The lower sub-figure of Figure 3 depicts the efficiency of the two schemes described above. We note that for $L \geq 3$, transmission+combining has significantly better efficiency. Further, as the number of antennas increases, the difference in slopes gets bigger.

We note, though, that there is a significant advantage to rate $1/2$ + repetition in terms of the block size (and thus latency), as its block size is $\rho(N)\left(2N\right)^{\ell-1}$, whereas the block size of transmission+combining is $\rho(N^2)\cdot 2^{\ell-1}$. Since $\rho(N)$ scales exponentially with N, this amounts to a very significant difference.

REFERENCES

[1] M.-T. Suer, C. Thein, H. Tchouankem, and L. Wolf, "Multi-connectivity as an enabler for reliable low latency communications—an overview," *IEEE Communications Surveys & Tutorials*, vol. 22, no. 1, pp. 156–169, 2019.

[2] Z. Li, L. Xiang, X. Ge, G. Mao, and H.-C. Chao, "Latency and reliability of mmWave multi-hop V2V communications under relay selections," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 9, pp. 9807–9821, 2020.

[3] B. Singh, Z. Li, O. Tirkkonen, M. A. Uusitalo, and P. Mogensen, "Ultra-reliable communication in a factory environment for 5G wireless networks: Link level and deployment study," in *2016 IEEE 27th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*. IEEE, 2016, pp. 1–5.

[4] P. Popovski, "Ultra-reliable communication in 5G wireless systems," in *1st International Conference on 5G for Ubiquitous Connectivity*. IEEE, 2014, pp. 146–151.

[5] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Transactions on Information theory*, vol. 45, no. 5, pp. 1456–1467, 1999.

[6] B. Singh, O. Tirkkonen, Z. Li, M. A. Uusitalo, and R. Wichman, "Selective multi-hop relaying for ultra-reliable communication in a factory environment," in *2016 IEEE 27th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*. IEEE, 2016, pp. 1–6.

[7] Y. Hu, C. Schnelling, M. C. Gursoy, and A. Schmeink, "Multi-relay-assisted low-latency high-reliability communications with best single relay selection," *IEEE Transactions on Vehicular Technology*, vol. 68, no. 8, pp. 7630–7642, 2019.

[8] S. Yang and J.-C. Belfiore, "Diversity of MIMO multihop relay channels," *arXiv preprint arXiv:0708.0386*, 2007.

[9] E. Domanovitz and U. Erez, "Diversity combining via universal dimension- reducing space–time transformations," *IEEE Transactions on Communications*, vol. 67, no. 3, pp. 2464–2475, 2019.

[10] B. Avraham, U. Erez, and E. Domanovitz, "Rate 1 quasi orthogonal universal transmission and combining for MIMO systems achieving full diversity," in *ICASSP 2021-2021 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2021, pp. 4875–4879.

[11] B. Avraham, "Dimension reduction for general MIMO systems," Master's thesis, Tel Aviv University, Tel Aviv, 2020.